Two-dimensional Effects on the CSR Interaction Forces for an Energy-Chirped Bunch

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3. Bunch Distribution Variation in a Chicane
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1. Introduction

- In JLAB FEL, when the bunch is fully compressed by the chicane, beam fragmentation is observed in the second arc which has not been reproduced by 1D CSR model.
(measurement done by D. Douglas, P. Evtushenko)

- There are some other unexplained observations, such as one emittance measurement (in CTFII) which shows strong dependence of emittance growth on transverse beta function inside the chicane.

- These motivate us to study the 2D CSR effect, as well as to investigate other possible causes.

Questions

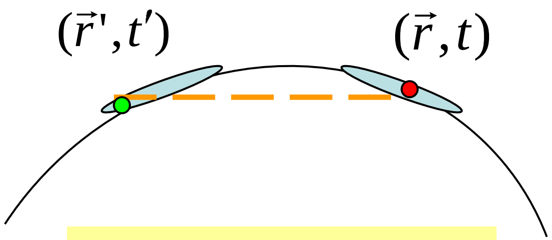
- In which parameter regime is the 1D model a good approximation ?

- How does bunch transverse size influence the CSR interaction forces?

Goal: full understanding of the CSR effect in bunch compression chicane so as to predict and control its adverse effect.

•It is hoped that a systematic analysis for a simple Gaussian case can provide some insight of the 2D effects and serve as benchmark for the 2D simulation.
(free space, unperturbed Gaussian bunch, circular orbit)

Overview of Basic Formulas for the CSR Interaction on a Curved Orbit



$$t' = t - \frac{|\vec{r} - \vec{r}\,'|}{c}$$

Retarded Potential (Lorentz gauge):

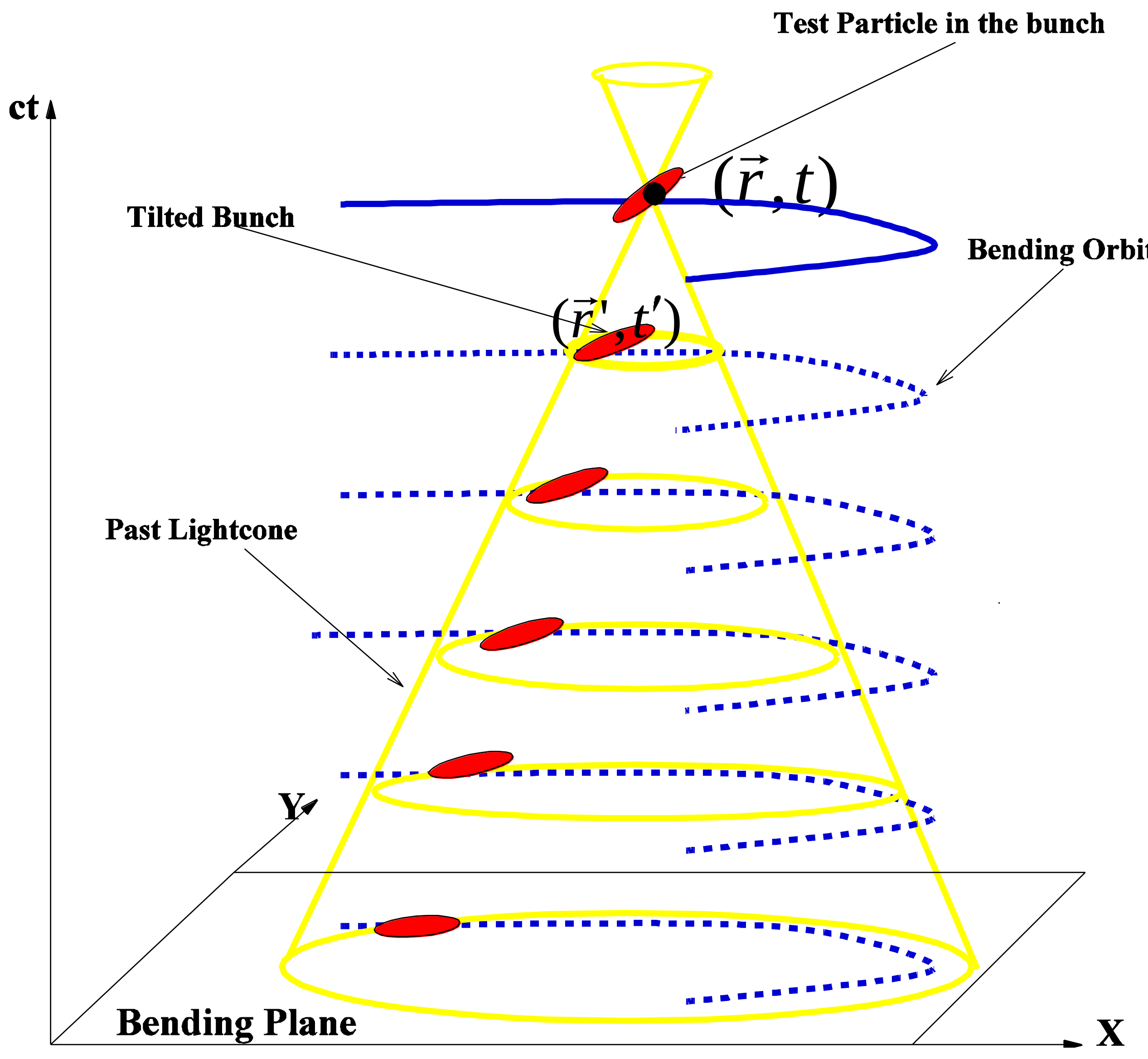
$$\begin{cases} \Phi(\vec{r}, t) = \displaystyle\int \frac{\rho(\vec{r}\,', t - |\vec{r} - \vec{r}\,'|/c)}{|\vec{r} - \vec{r}\,'|} d^3\vec{r}\,' \\ \vec{A}(\vec{r}, t) = \displaystyle\int \frac{\vec{J}(\vec{r}\,', t - |\vec{r} - \vec{r}\,'|/c)}{|\vec{r} - \vec{r}\,'|} d^3\vec{r}\,' \end{cases}$$

(assuming continuous charge distribution)

Electromagnetic Fields: $$\vec{E}(\vec{r}, t) = -\vec{\nabla}\Phi - \frac{\partial \vec{A}}{c\partial t}, \quad \vec{B}(\vec{r}, t) = \nabla \times \vec{A}$$

Lorentz Force On each electron: $$\vec{F} = e(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$$

Illustration of CSR interaction for a 2D bunch



2. Previous 1D and 2D Results of Effective CSR Forces

The CSR interaction influences both the longitudinal and transverse particle dynamics via effective CSR forces (Derbenev, 1996)

Electromagnetic Fields: $\vec{E}(\vec{r},t) = -\vec{\nabla}\Phi - \frac{\partial \vec{A}}{c\partial t}, \quad \vec{B}(\vec{r},t) = \nabla \times \vec{A}$

Lorentz Force In dynamic eqn: $\frac{d(\gamma m\vec{v})}{dt} = e(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$

Rate of kinetic energy loss

$$\frac{d(\gamma mc^2)}{cdt} = e\vec{E}\cdot\frac{\vec{v}}{c} = -e\frac{d\Phi}{cdt} \underbrace{- e\left(\frac{\partial\Phi}{c\partial t} - \vec{\beta}\cdot\frac{\partial\vec{A}}{c\partial t}\right)}_{F_s^{\text{eff}}}$$

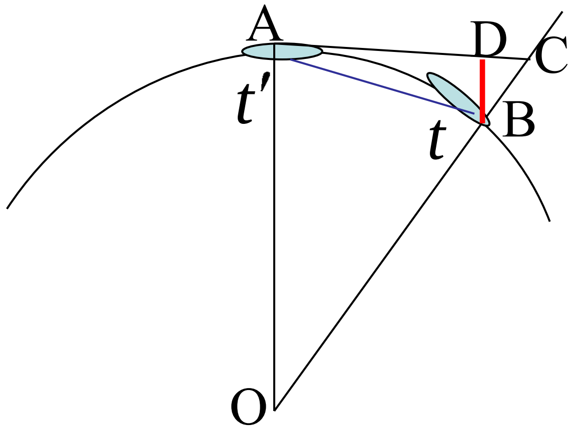
$$\frac{d(\gamma mc^2 + e\Phi)}{cdt} = F_s^{\text{eff}} \quad \text{or} \quad \frac{dH}{dt} = \frac{\partial H}{\partial t}$$

Effective longitudinal CSR force

Here F_s^{eff} plays an important role in influencing particle transverse dynamics

$$\gamma(t)mc^2 = -e\Phi(t) + (\gamma mc^2 + e\Phi)_{t=0} + \int_0^t F_s^{\text{eff}}(t')cdt'$$

1D Assumption and Result (Derbenev, 1996)



Characteristic distance:

$$AB \approx (24\sigma_z R^2)^{1/3}, \qquad DB \approx 2(9\sigma_z^2 R)^{1/3}$$

1D assumption:

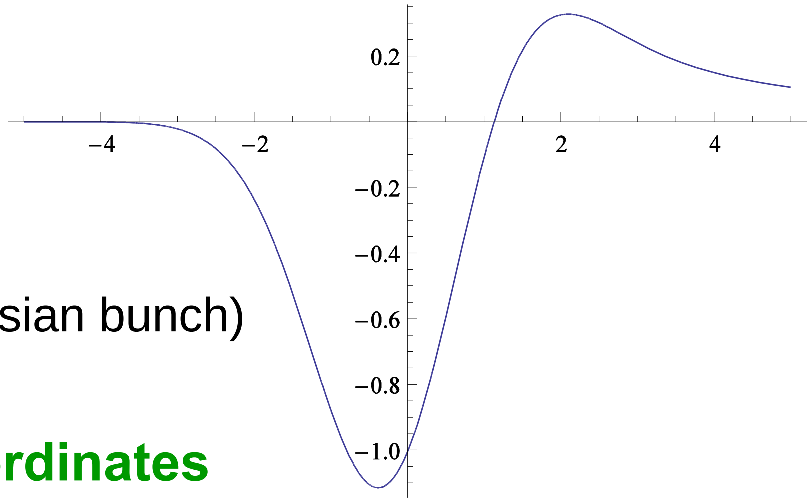
$$\frac{\sigma_x}{DB} << 1, \quad \text{or} \quad \alpha = \frac{\sigma_x}{(\sigma_z^2 R)^{1/3}} << 1$$

Effective Force in Steady-state:

$$F_s^{\text{eff}}(z) = -\frac{2Ne^2}{3^{1/3}R^{2/3}} \int_{-\infty}^{z} \frac{dz}{(z-z')^{1/3}} \frac{\partial \lambda(z')}{\partial z'}$$

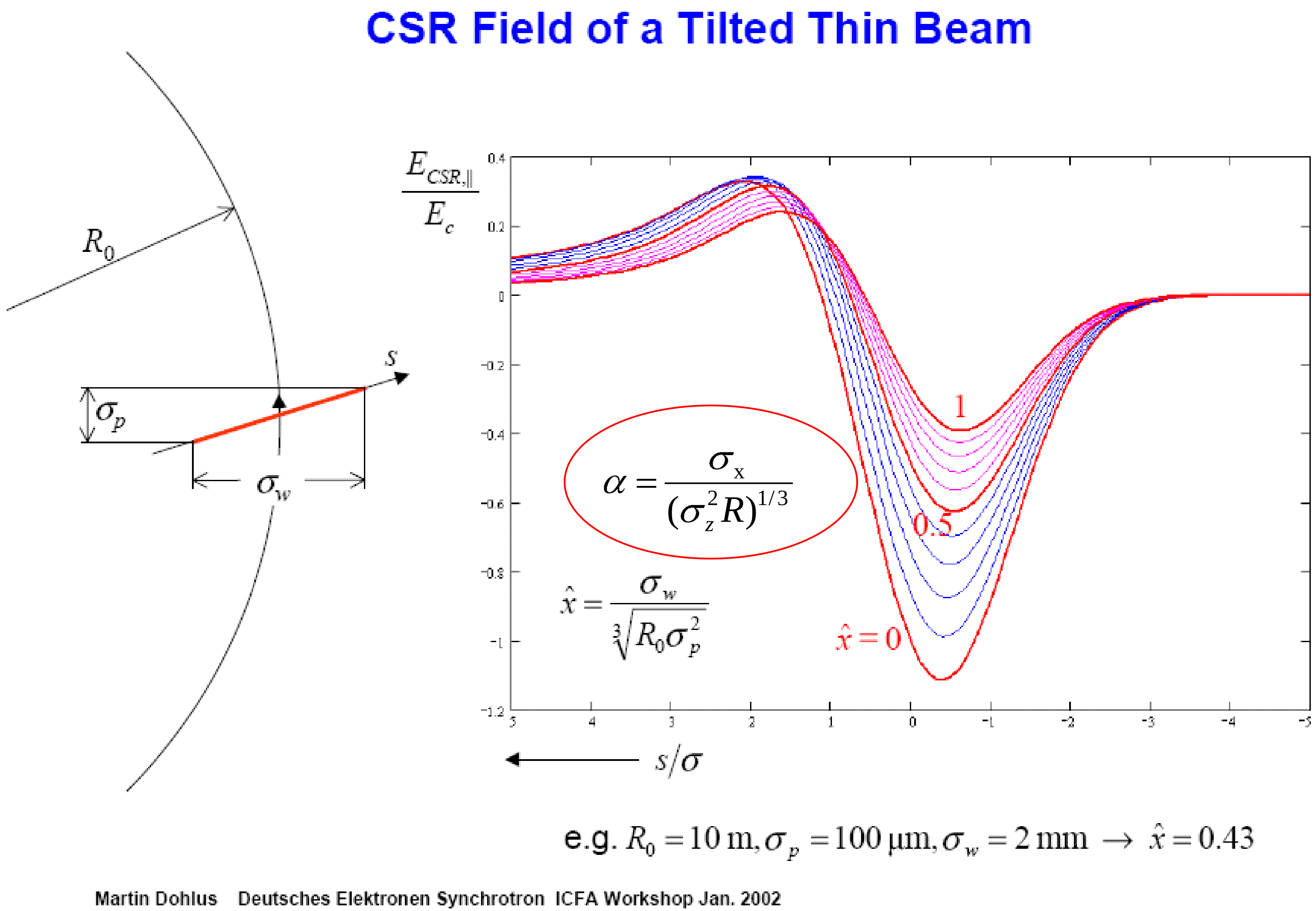
$$F_s^{\text{eff}}(z) = \frac{2N_p e^2}{\sqrt{2\pi}\, 3^{1/3} \sigma_z^{4/3} R^{2/3}} f(z) \qquad \text{(Gaussian bunch)}$$

f(z):form factor



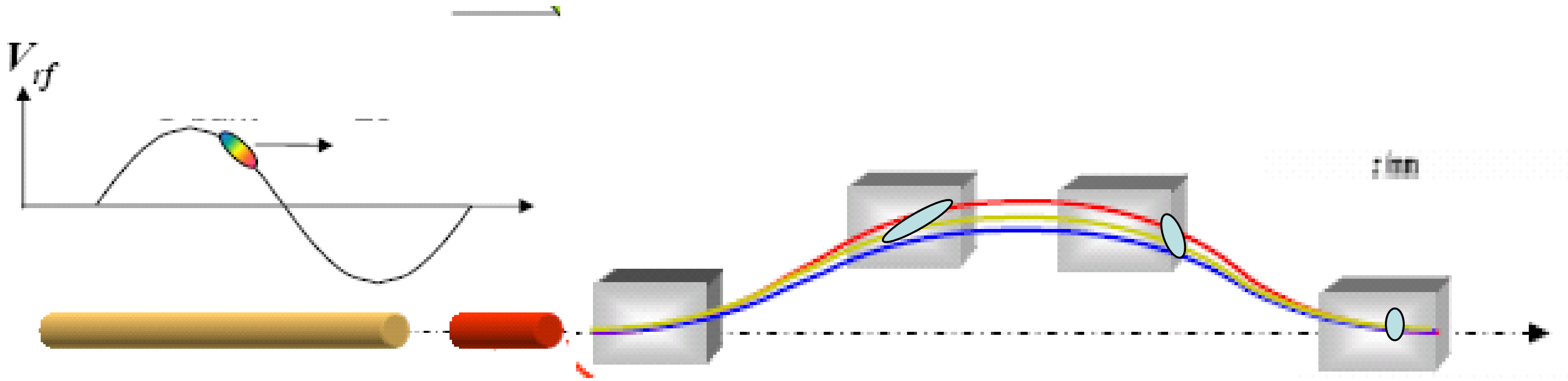
Feature: no dependence on particles' x coordinates

Previous 2D Results for a Gaussian Bunch (Dohlus,2002)



- As the deflection gets bigger, the CSR force is smaller in amplitude as compared to the rigid-bunch result (with the same projected bunch length)

3.Bunch Distribution Variation in a Chicane



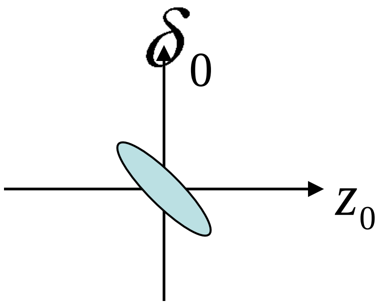
(Drawing from P. Krejcik, SLAC)

Bunch compression is carried out via

- Initial $\delta - z$ correlation
- $x - \delta$ correlation in dispersive regions

This gives x-z correlation=> bunch deflection in dispersive regions

- Linear Optic Transport from s_0 to s

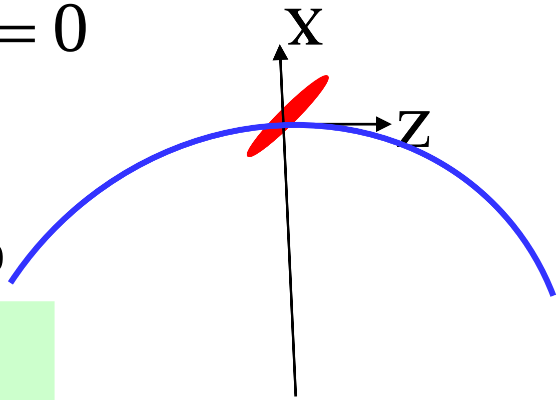


$$\begin{cases} x(s) = R_{11}(s)x_0 + R_{12}(s)x_0' + R_{16}(s)\delta_0 \\ z(s) = R_{51}(s)x_0 + R_{52}(s)x_0' + z_0 + R_{56}(s)\delta_0 \end{cases}$$

with initial energy chirp $\delta_0 = u\, z_0 + \delta_{\mathrm{un}}$

- Zero uncorrelated spread $x_0 = x_0' = \delta_{\mathrm{un}} = 0$

$$\begin{cases} x(s) = R_{16}(s)\,(uz_0) \\ z(s) = z_0 + R_{56}(s)\,(u\, z_0) = (1 + u\, R_{56}(s))z_0 \end{cases}$$



Compression factor $\mathrm{R}_{55}(s) = 1 + uR_{56}(s)$

slope of deflection $\xi(s) = \dfrac{x(s)}{z(s)} = \dfrac{uR_{16}(s)}{1 + uR_{56}(s)}$

u=0 for bunch with no deflection.

- Example of the Benchmark Chicane

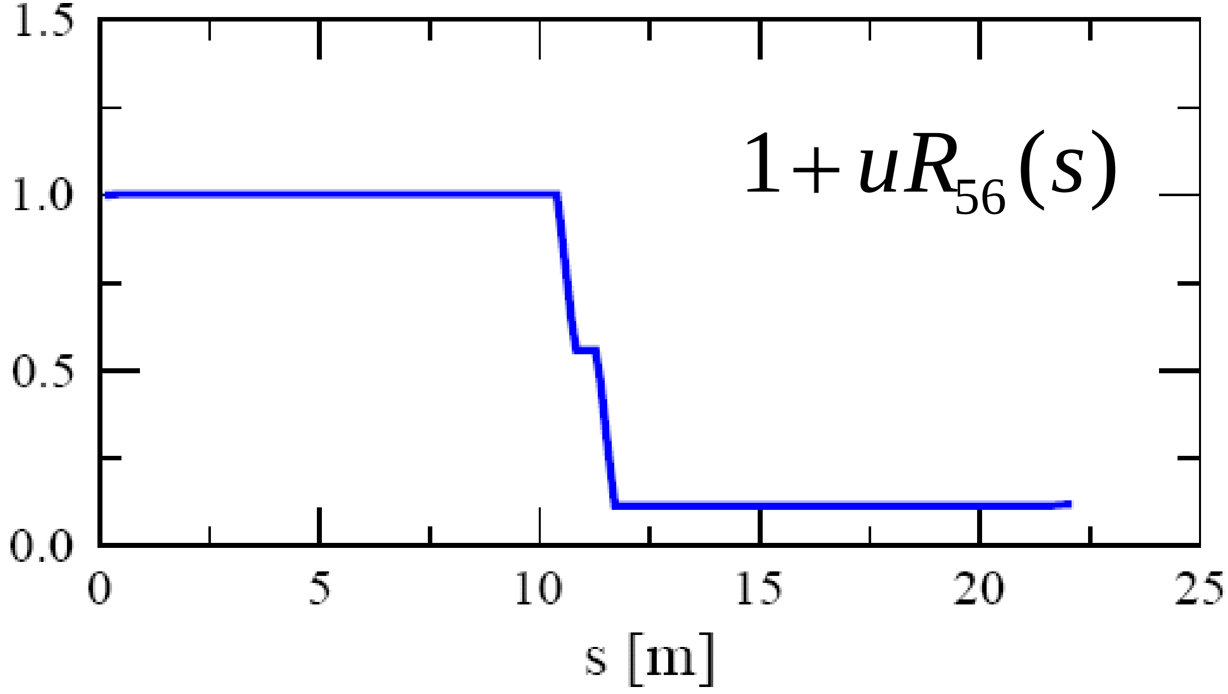
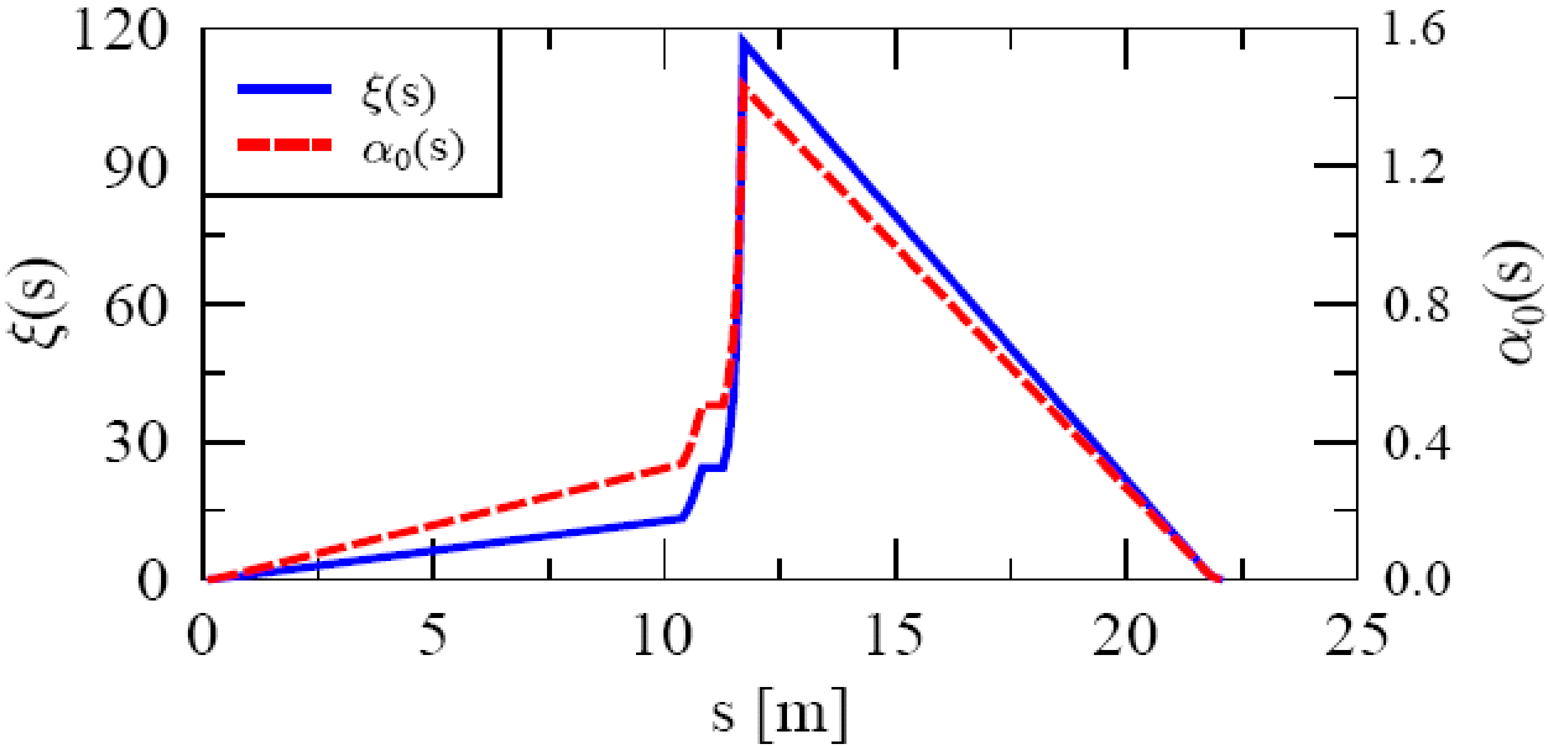


FIG. 1: Compression factor, $\mathcal{R}_{55}(s) = 1 + uR_{56}(s)$, vs. s for $u = -40$ m^{-1}.

- Example of the Benchmark Chicane (assuming zero initial emittance and energy spread)



- Studies using 1D CSR interaction model is semi-self-consistent (for CSR force calculation: projecting 2D bunch on design orbit and assuming rigid-bunch at retarded time)

$$\frac{\partial \rho}{\partial s} - \frac{x}{R}\frac{\partial \rho}{\partial z} + \theta \frac{\partial \rho}{\partial x} + \left(-k_\beta^2 x + \frac{p}{R}\right)\frac{\partial \rho}{\partial \theta} = \frac{r_e}{\gamma}\frac{\partial \rho}{\partial p}\int dz' \, W(z - z', s) n(z', s),$$

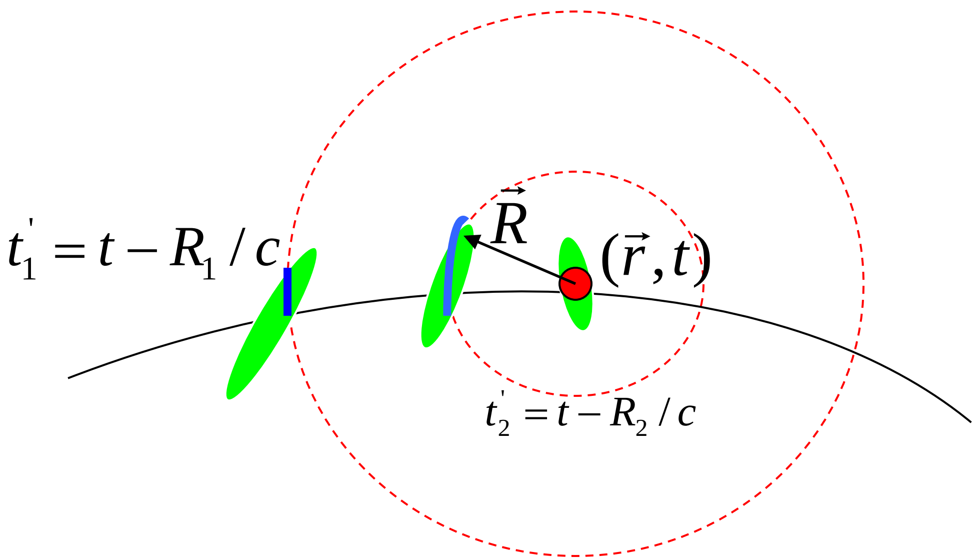
for $W(z) = \frac{2}{(3R^2)^{1/3}} \frac{\partial}{\partial z} z^{-1/3} \quad (z > 0)$ or $Z(k) = -iA\frac{k^{1/3}}{R^{2/3}} \quad (A = 1.63i - 0.94)$

- Need to include evolution of bunch x-z distribution in the CSR force calculation

4. Retardation for 2D Interaction

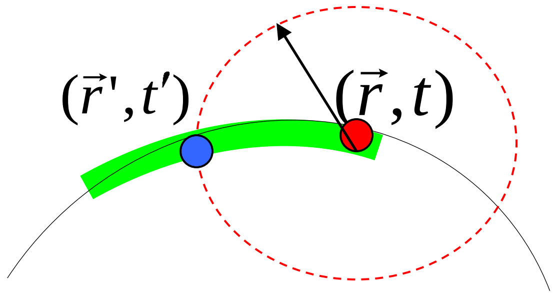
The CSR force is influenced by the bunch x-z deflection mainly through retardation → identify the source particles at retarded time

•**2D bunch:** $$\Phi(\vec{r},t)=\int\frac{\rho(\vec{r}\,',t-|\vec{r}-\vec{r}\,'|/c)}{|\vec{r}-\vec{r}\,'|}d^2\vec{r}\,'\xrightarrow{\vec{r}\,'-\vec{r}=\vec{R}}\int\frac{\rho(\vec{r}+\vec{R},t-R/c)}{R}RdRd\theta$$

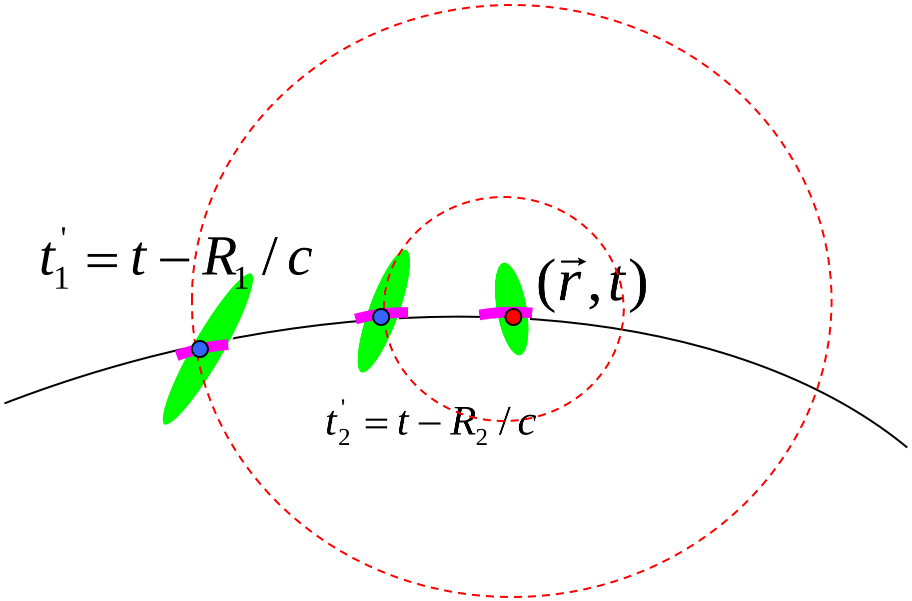


For each t', the source particles are on a cross-section with x-s dependence

- 1D line bunch



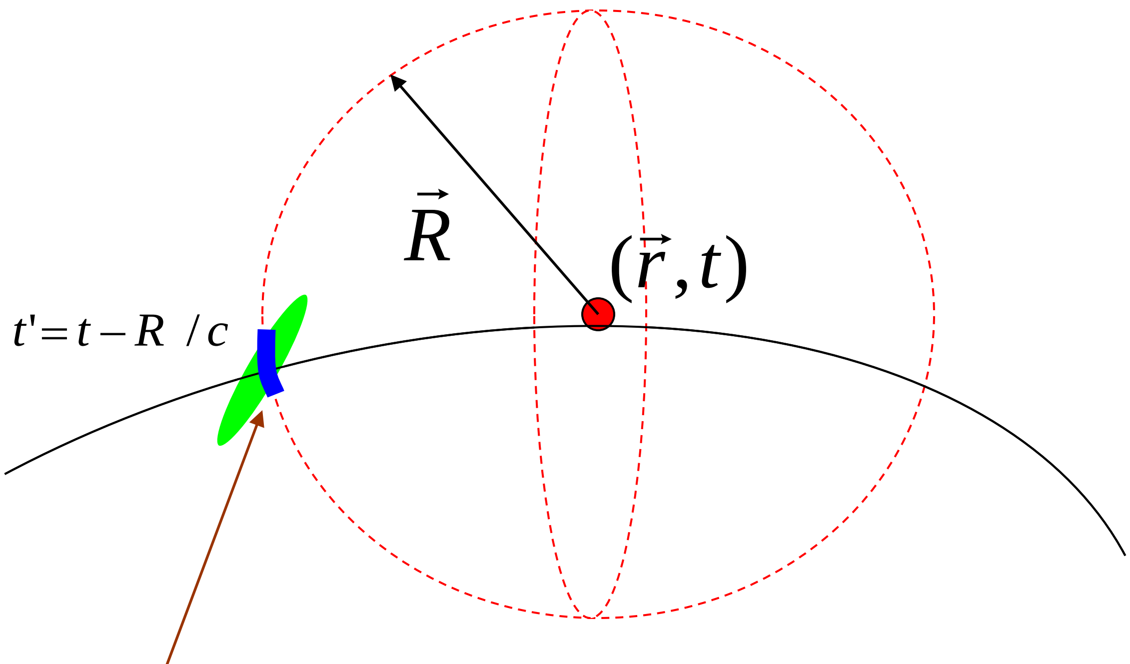
Retardation for a 2D bunch projected on the design orbit, assuming longitudinal distribution frozen in the past :



3D Retardation

(illustration not to scale)

$$\Phi(\vec{r},t) = \int \frac{\rho(\vec{r}\,',t-|\vec{r}-\vec{r}\,'|/c)}{|\vec{r}-\vec{r}\,'|} d^2\vec{r}\,' \xrightarrow{\vec{r}\,'-\vec{r}=\vec{R}} \int \frac{\rho(\vec{r}+\vec{R},t-R/c)}{R} R^2 dR \sin\theta d\theta d\phi$$



Intersection of the sphere (light cone for 4D space-time) with the 3D bunch

Because of dispersion, transverse size is often much bigger than the vertical size, so 2D is often a good approximation.

5. CSR Forces for a 2D Bunch Self Interaction

Assumptions used in the analytical study of 2D CSR force:

- Perfect initial Gaussian distribution in transverse and longitudinal phase space
- Initial linear energy chirp
- Transport by linear optics from external field, bunch is unperturbed by the CSR self-interaction.
- All CSR interactions take place inside bend

$$\begin{cases} \Phi(\vec{r},t) = \displaystyle\int \frac{\rho(\vec{r}\,',t-|\vec{r}-\vec{r}\,'|/c)}{|\vec{r}-\vec{r}\,'|} d^3\vec{r}\,' \\ \vec{A}(\vec{r},t) = \displaystyle\int \frac{\vec{J}(\vec{r}\,',t-|\vec{r}-\vec{r}\,'|/c)}{|\vec{r}-\vec{r}\,'|} d^3\vec{r}\,' \end{cases}$$

Analytical approach:

- For CSR force on each test particle in the bunch, solve 2D retardation relation analytically
- Identify the source particle which emitted field at a source location.
- Find the phase space distribution of the source particle by propagating initial Gaussian distribution via linear optics to the source location
- For CSR force on a test particle, integrate contribution from all source particles in the initial distribution.

Thin bunch → analytical results

General cases → numerical integration of integrands which is an analytical expression involving retardation, linear optics and initial Gaussian distribution

•Analytical Result of Effective Longitudinal CSR Force

(for a thin Gaussian bunch)

$$\tilde{F}_H(\tilde{z}_0,\alpha) \simeq \frac{2Nr_e}{\gamma_0 3^{1/3}\sqrt{2\pi}[\sigma_z(s)]^{4/3}|R_0|^{2/3}} I(\tilde{z}_0,\alpha),$$

$$I(\tilde{z}_0,\alpha) = \frac{3^{1/3}}{4}\int_0^\infty d\Delta\tilde{s}\frac{\Delta\tilde{s}}{\Lambda_1(\Delta\tilde{s},\alpha)}(\tilde{z}_0-\Delta\tilde{z}_0)\exp\left[-\frac{(\tilde{z}_0-\Delta\tilde{z}_0)^2}{2}\right]$$

$$\Lambda_1(\Delta\tilde{s},\alpha) = \left(1+\frac{\alpha\Delta\tilde{s}}{2}\right)\sqrt{\left(1+\frac{\alpha\Delta\tilde{s}}{2}\right)^2+\frac{(\alpha\Delta\tilde{s})^2}{12}}.$$

$$\alpha = \frac{\sigma_x}{(\sigma_z^2 R)^{1/3}}$$

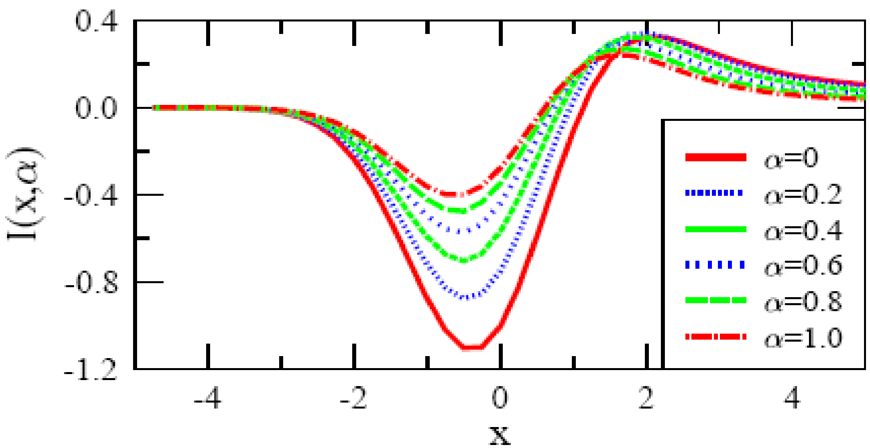
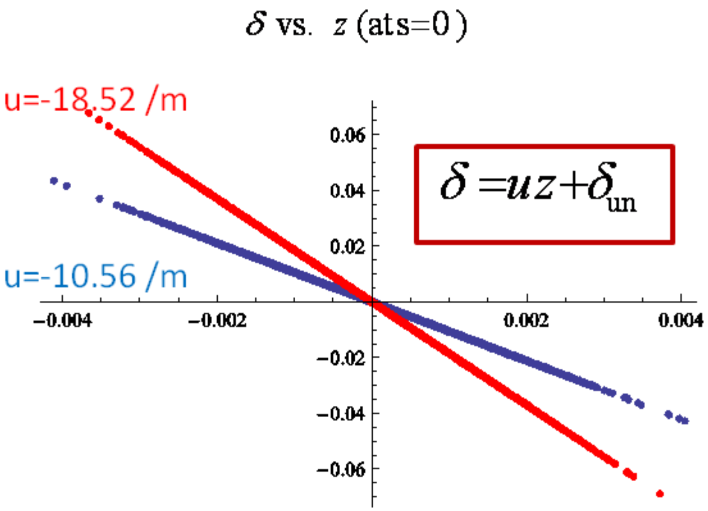


FIG. 2: $I(x,\alpha)$ vs. x for various α given by Eq. (122).

agree with Dohlus' Trafic4 result

Initial energy chirp of the bunch at entrance of the chicane



The Model Chicane : total path length=2m

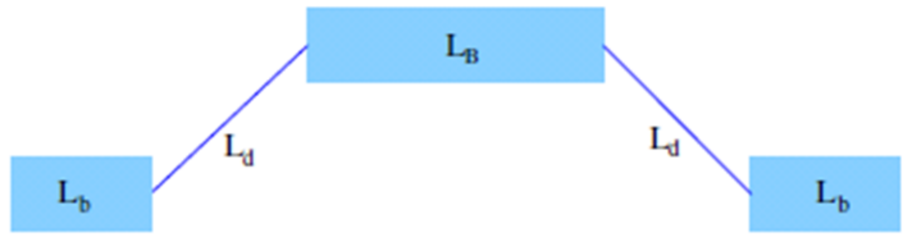
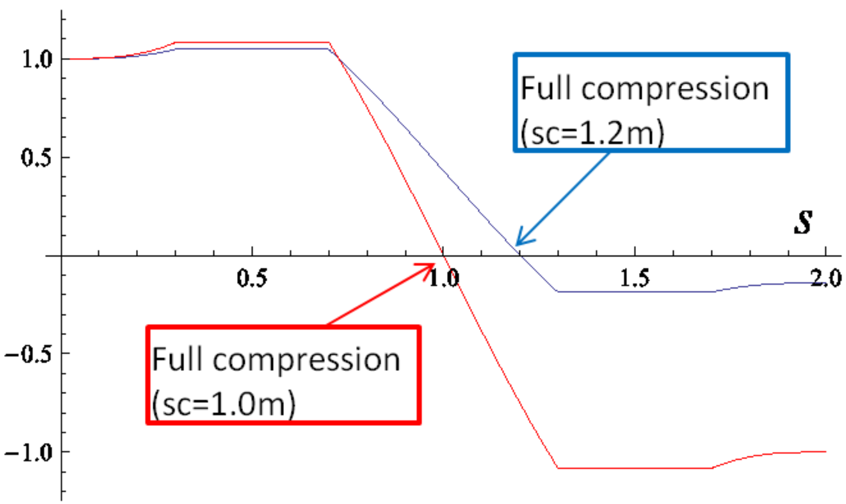
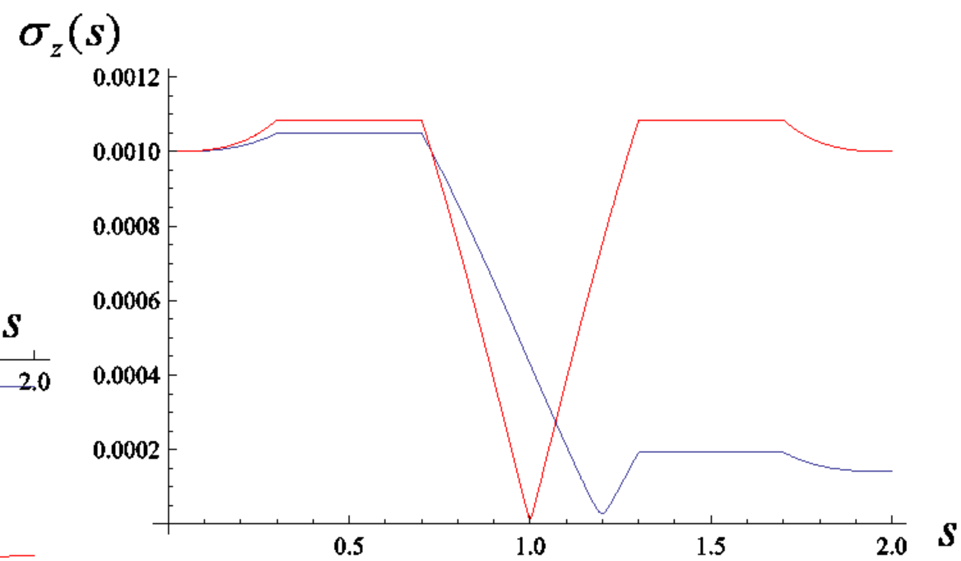


FIG. 3. (Color) Model chicane: $R = 1$ m, $L_b = 0.3$ m, $L_B = 0.6$ m, and $L_d = 0.4$ m.

Bunch compression factor vs s



Bunch length $\sigma_z(s)$ vs s



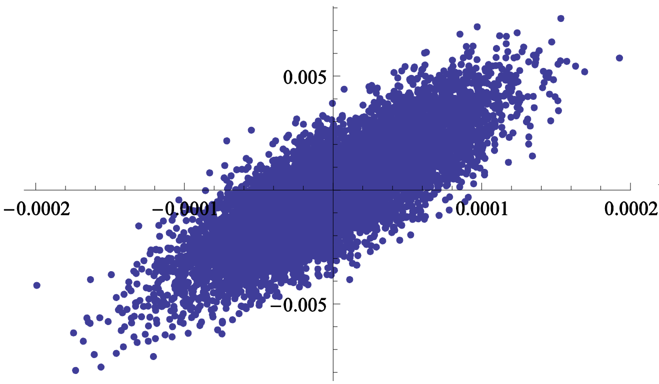
Initial beam parameters

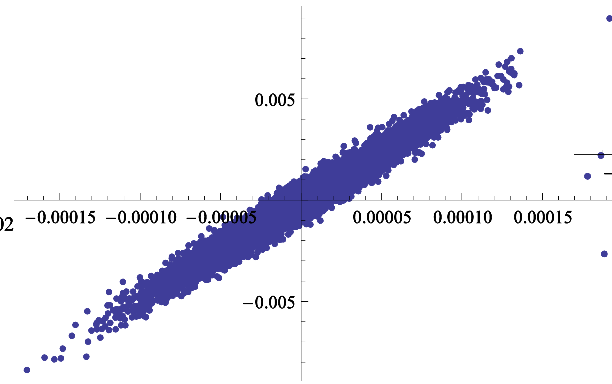
$E = 70\ \text{MeV}, \qquad \beta_x = 5\,\text{m},\ \alpha_x = 1$

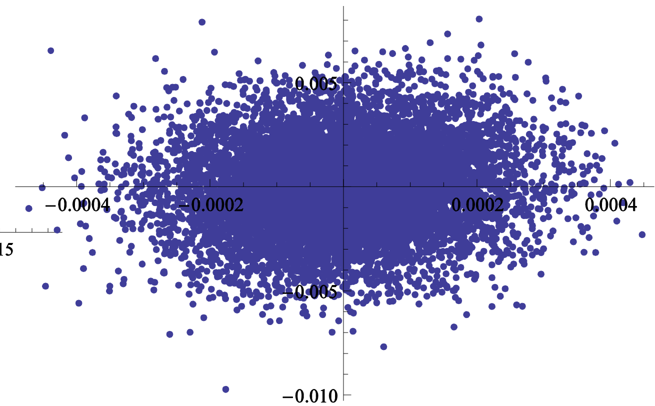
For each initial energy chirp, u=-10.56/m and u=-18.52/m, we studied five cases of normalized emittance and uncorrelated energy spread

thinner :	$\varepsilon_x = 0.1\ \mu m$	$\delta_{\text{un}} = 10^{-5}$
thin :	$\varepsilon_x = 0.1\ \mu m$	$\delta_{\text{un}} = 10^{-4}$
regular :	$\varepsilon_x = 1.0\ \mu m$	$\delta_{\text{un}} = 10^{-4}$
thick :	$\varepsilon_x = 1.0\ \mu m$	$\delta_{\text{un}} = 10^{-3}$
thicker :	$\varepsilon_x = 10\ \mu m$	$\delta_{\text{un}} = 10^{-3}$

$$z(s) = R_{51}(s)x_0 + R_{52}(s)x_0' + z_0 + R_{56}(s)\delta_0 \qquad (\delta_0 = uz_0 + \delta_{un})$$
$$= R_{51}(s)x_0 + R_{52}(s)x_0' + (1 + uR_{56}(s))z_0 + R_{56}(s)\delta_{un}$$







regular bunch

$\varepsilon_{nx} = 1\,\mu\text{m}$

$\sigma_H = 10^{-4}$

Very thin bunch

$\varepsilon_{nx} = 0.1\,\mu\text{m}$

$\sigma_H = 10^{-5}$

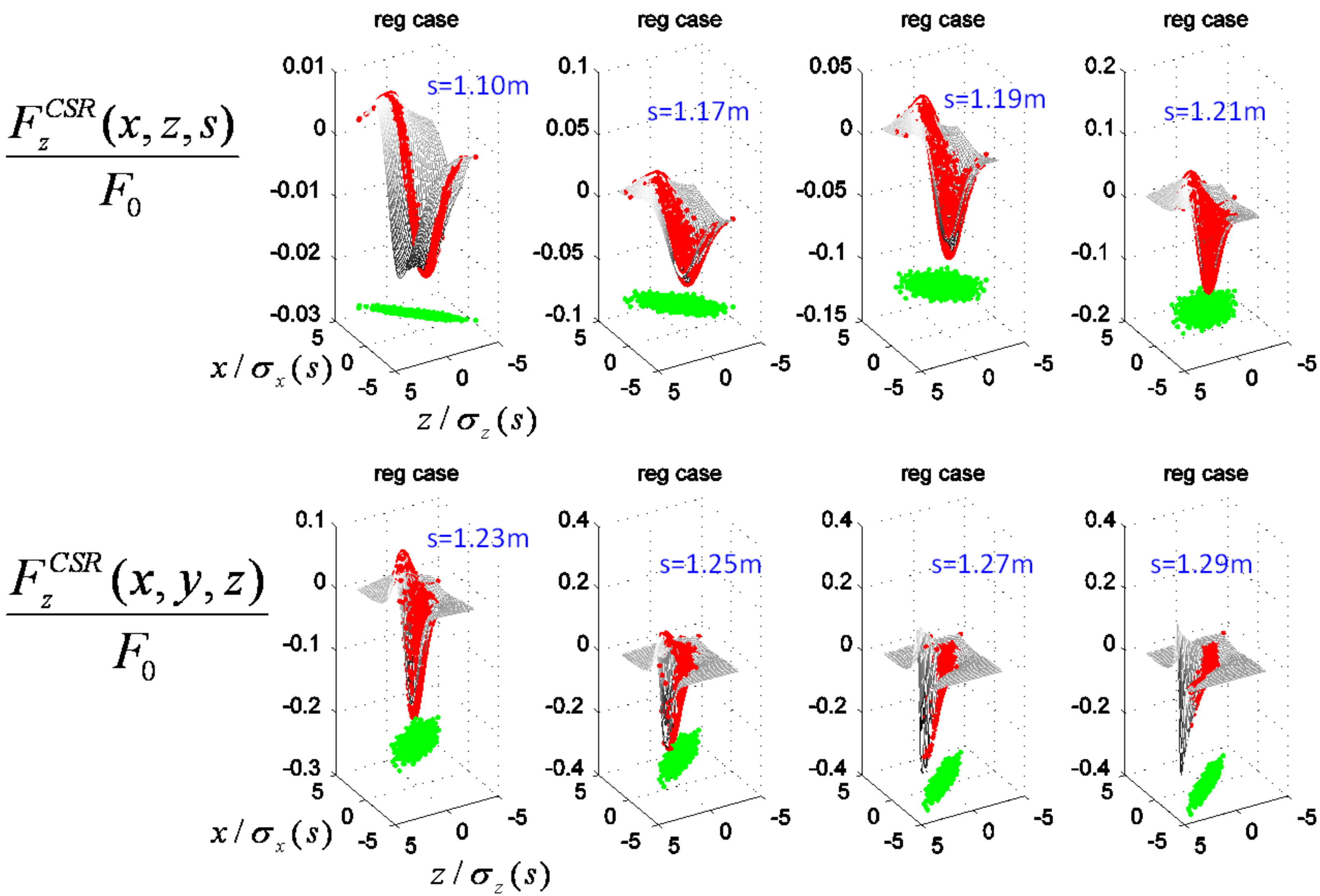
Very thick bunch

$\varepsilon_{nx} = 10\,\mu\text{m}$

$\sigma_H = 10^{-3}$

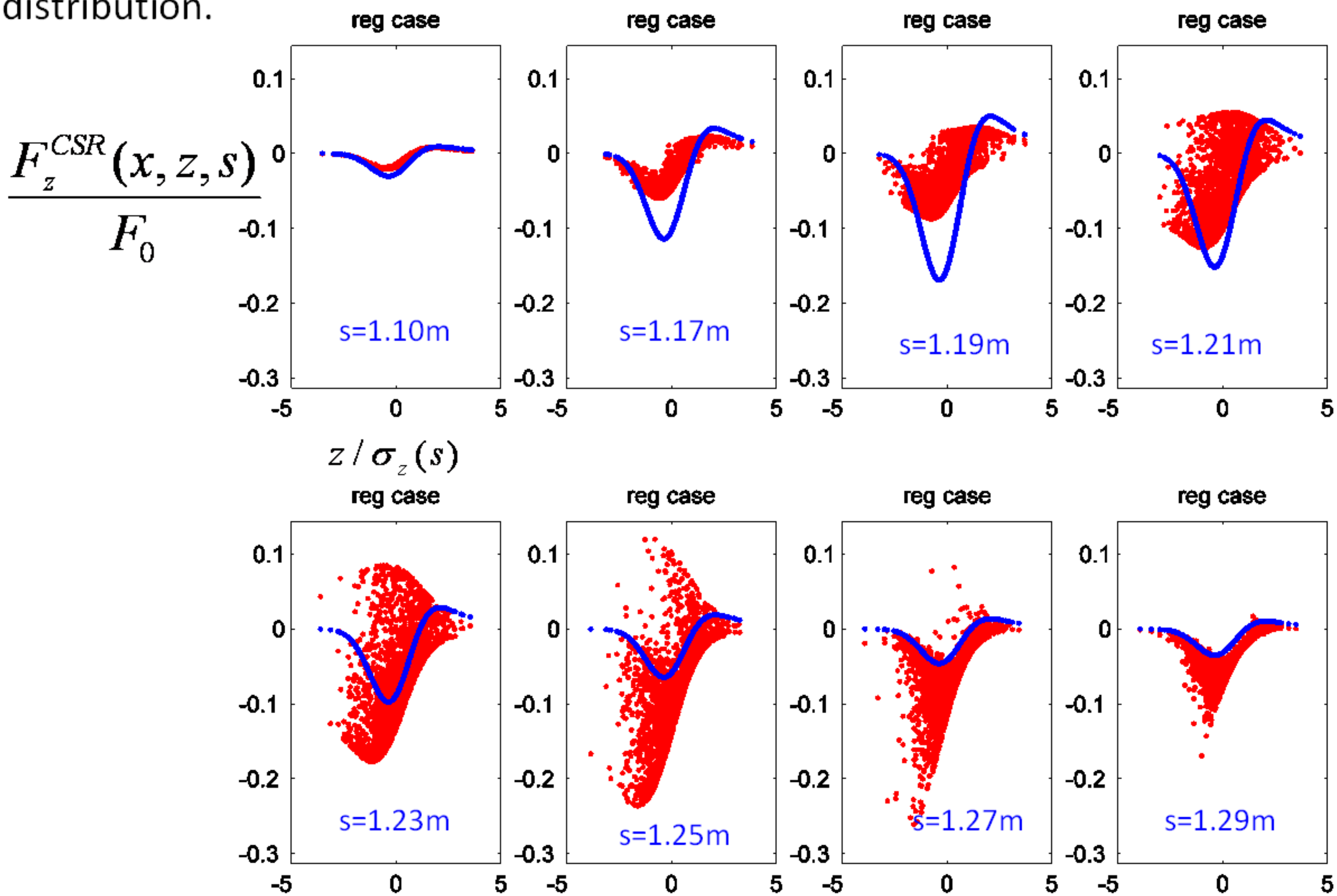
u=-10.56/m (sc=1.2m), regular case

2D CSR force on the x-z grid (grey) surrounding the bunch at various path length. The green is the footstep of the bunch in the x-z plane, and red is the 2D CSR force on the bunch.



u=-10.56/m (sc=1.2m), regular case

2D CSR force on the bunch is a function of both x and z. But the 1D model gives CSR force as a function of z only. Here we plot the 2D results of the CSR force as a function of z (red), the spread is due to the x-dependence. The blue curve is the CSR force based on 1D model using the projected longitudinal density distribution.



u=-10.56/m (sc=1.2m), regular case

As in previous slide, for a given path length s, 2D CSR force on each particles in the bunch has different amplitude depending on the x and z coordinates of the particle. Here we plot the maximum and minimum CSR force on the bunch as a function of s. The average of CSR force on all the particles in the bunch indicates the energy loss by the bunch. The 2D results are in red and 1D results are in green, which is based on the rigid-bunch model with the same projected bunch length.

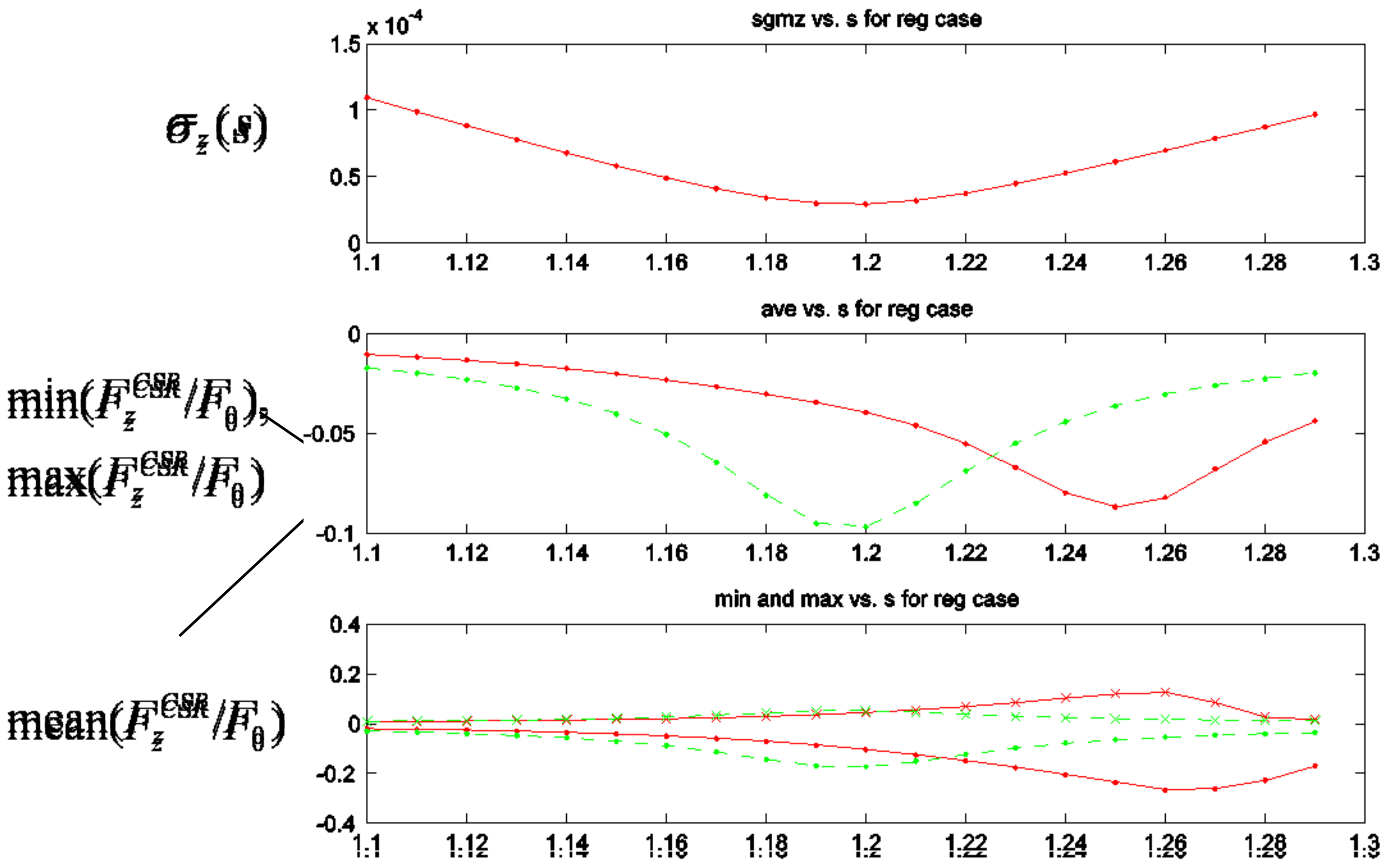
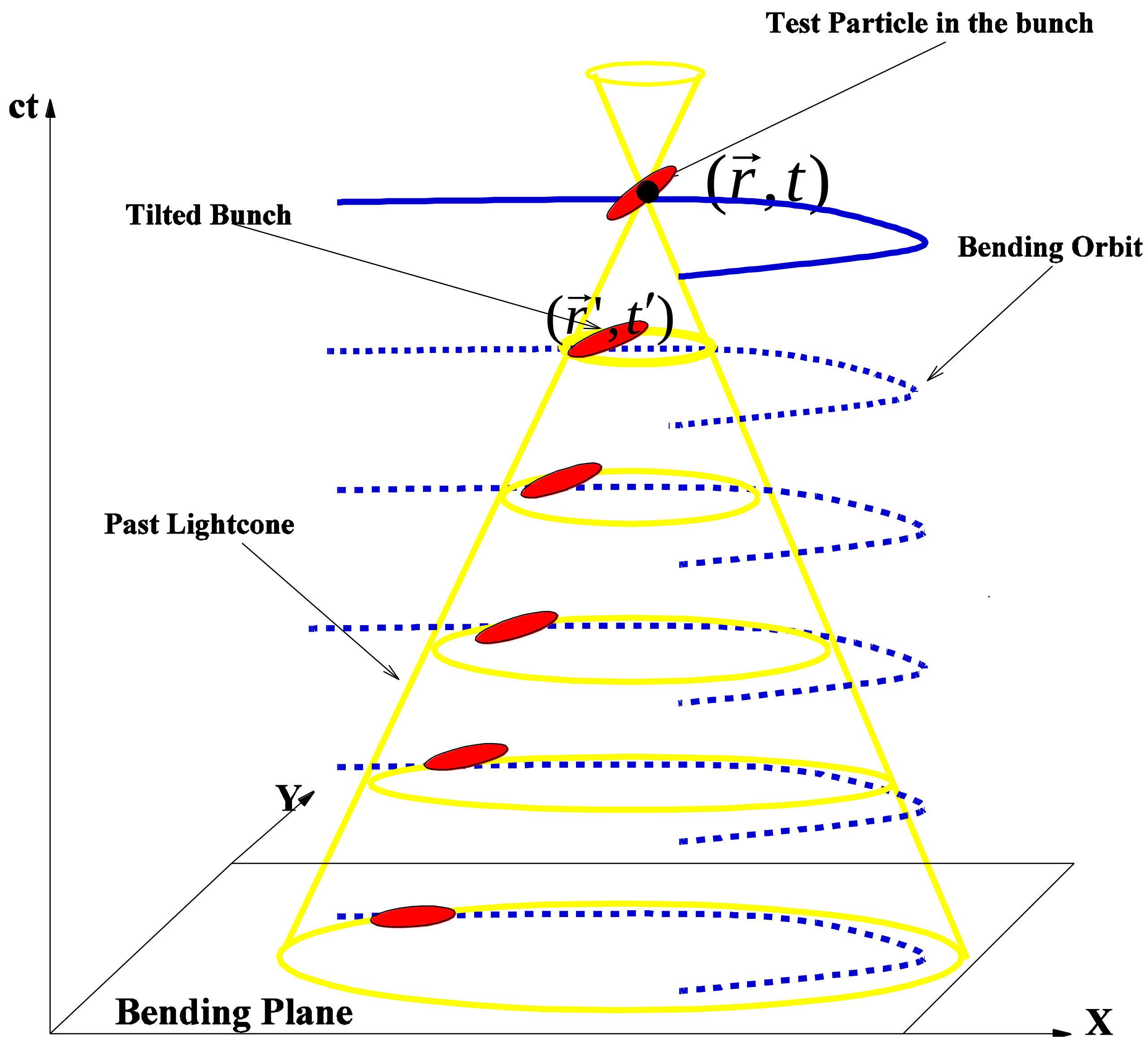
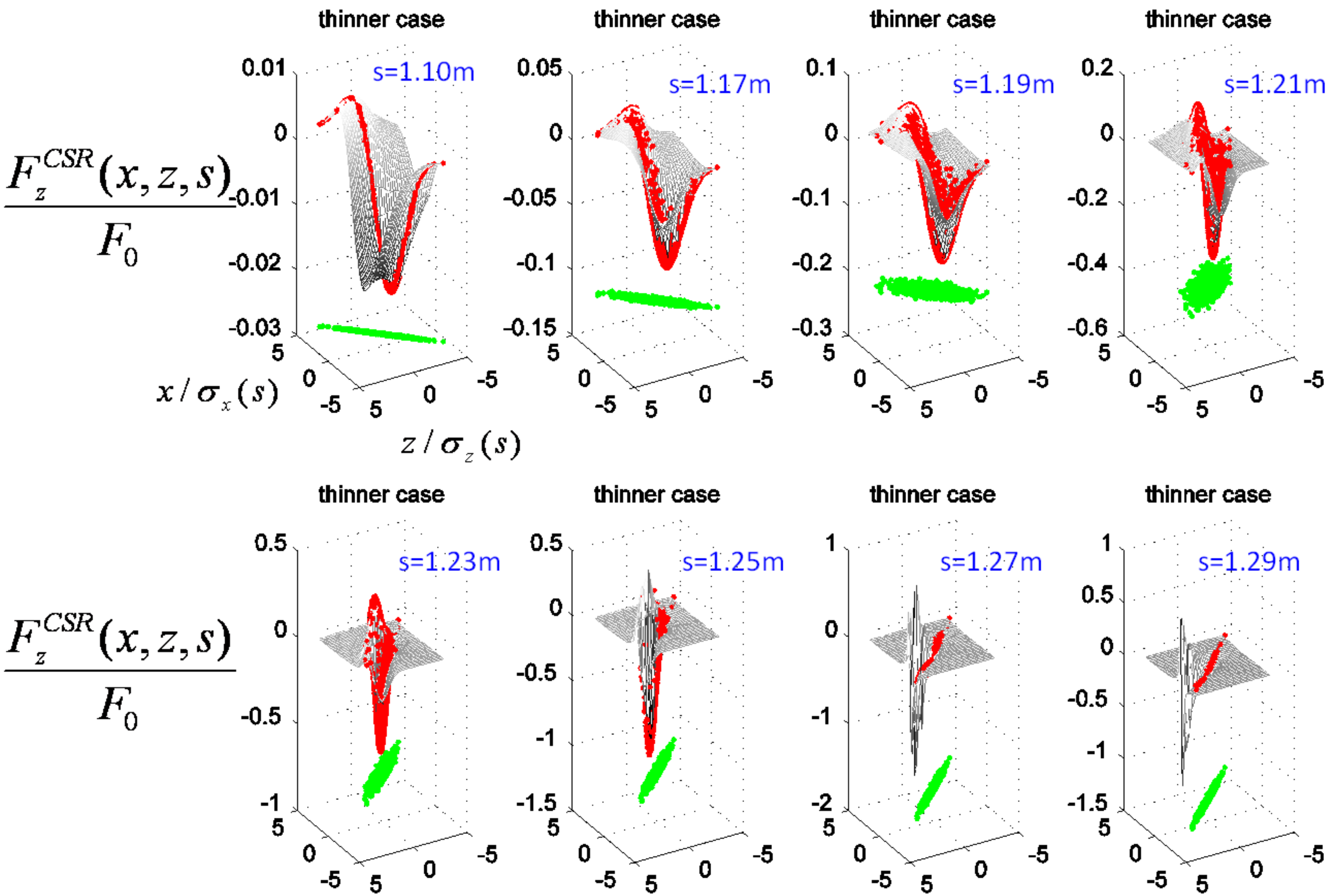


Illustration of CSR interaction for a 2D bunch

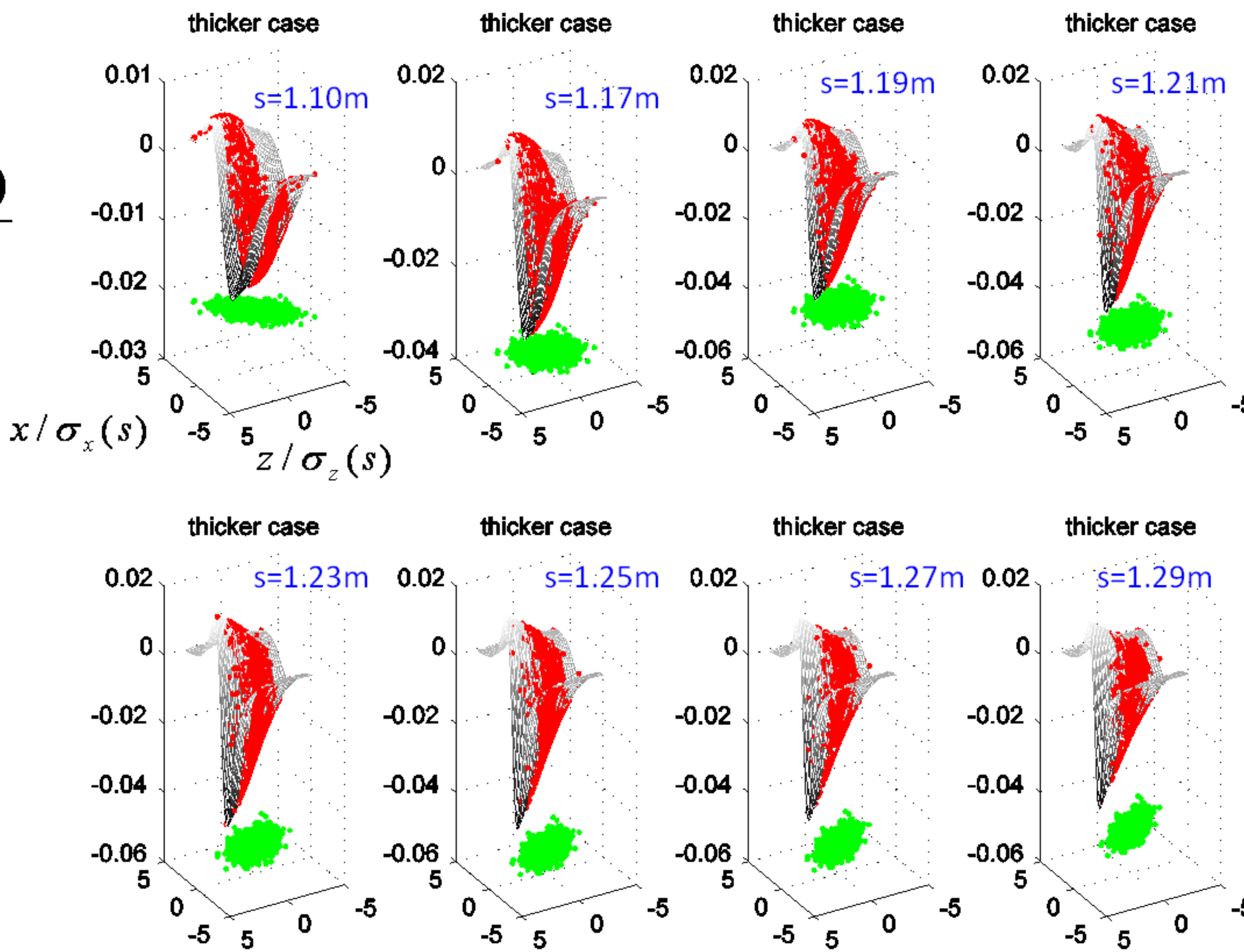


u=-10.56/m (sc=1.2m), thinner case

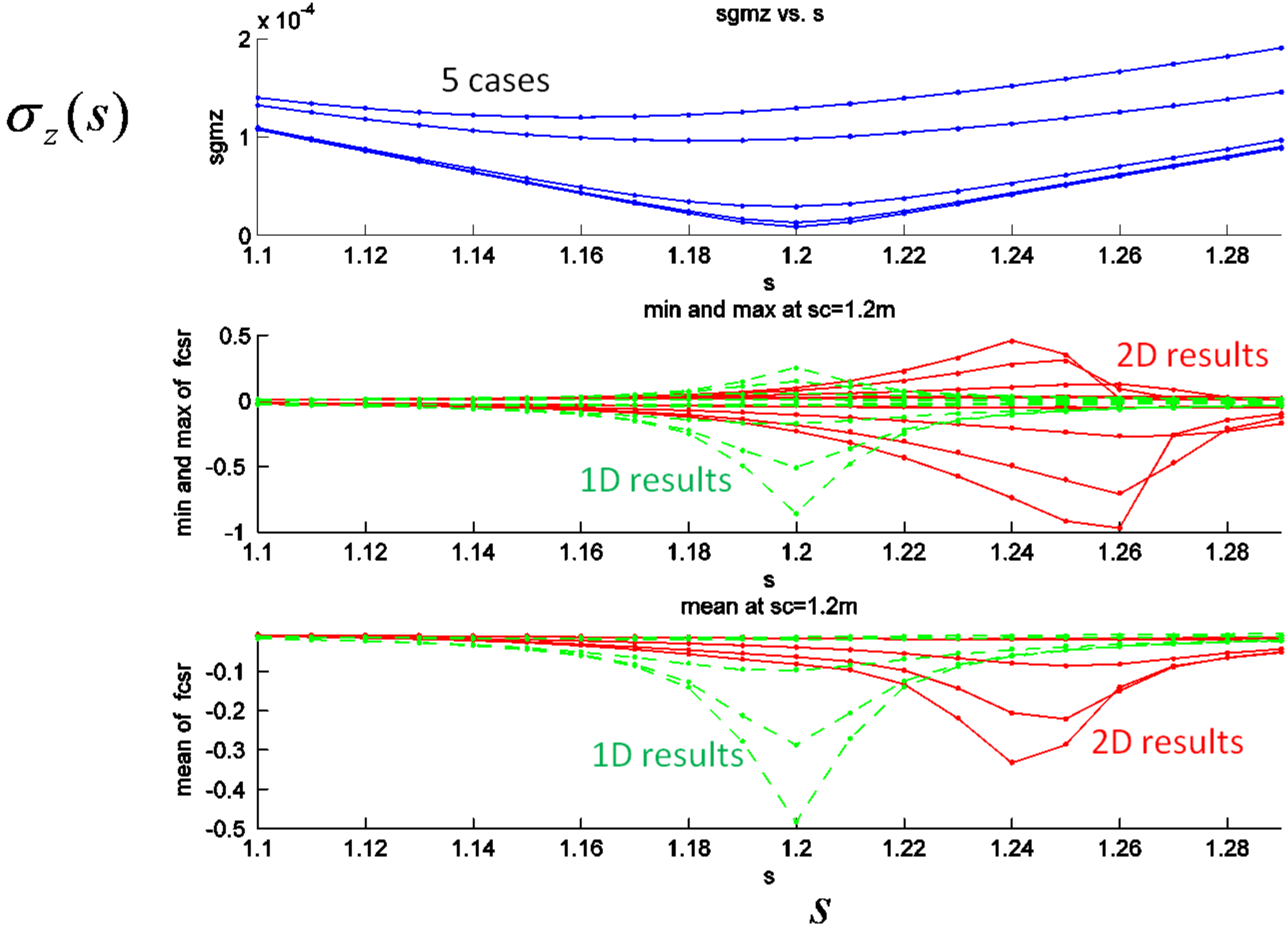


u=-10.56/m (sc=1.2m), thicker case

$$\frac{F_z^{CSR}(x,z,s)}{F_0}$$



Results for all 5 cases around full compression (u=-10.56/m, sc=1.2m)



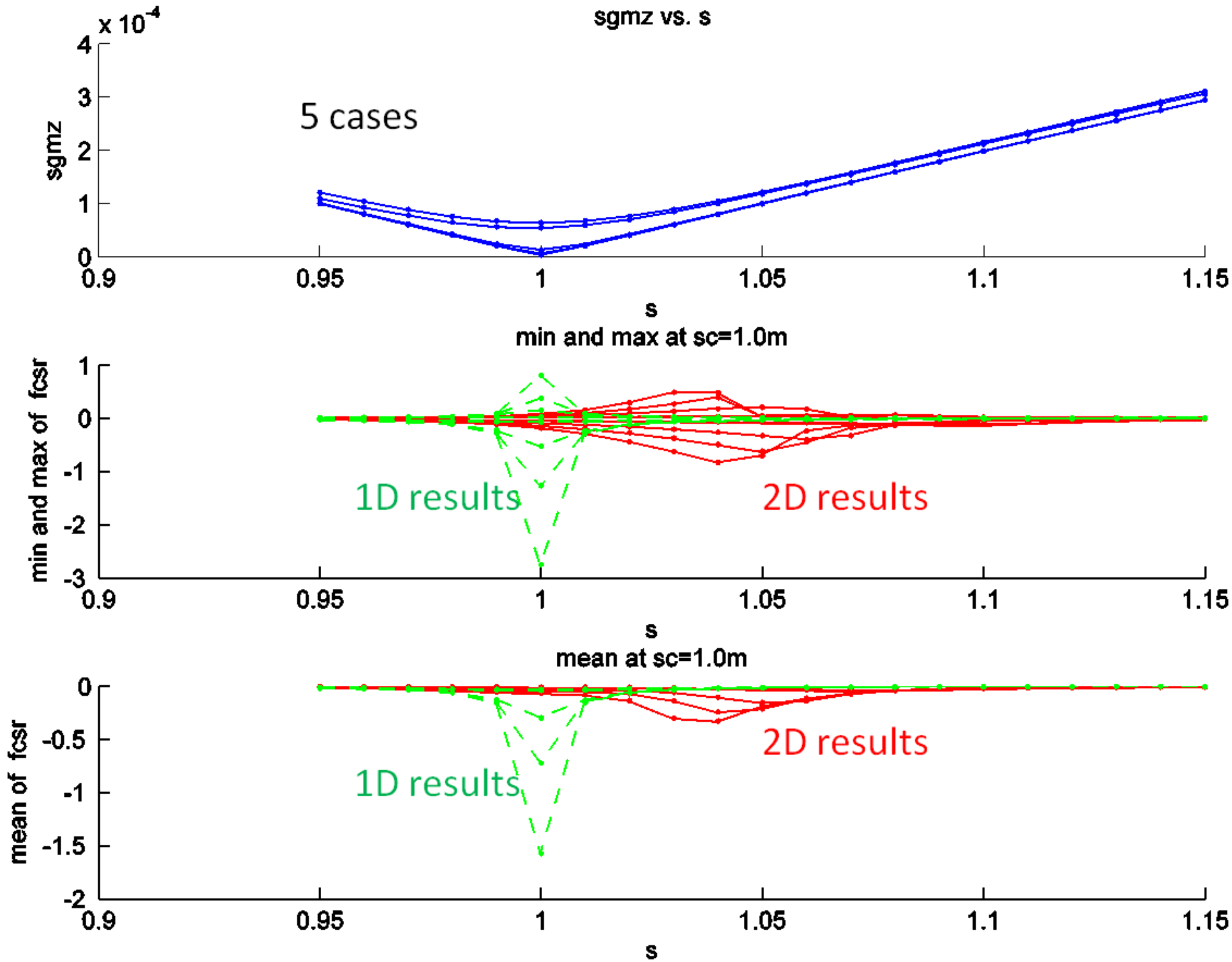
Results for all 5 cases around full compression (u=-18.52/m, sc=1.0m)

$\sigma_z(s)$

$\min(F_z^{CSR}/F_0)$,

$\max(F_z^{CSR}/F_0)$

$\text{mean}(F_z^{CSR}/F_0)$



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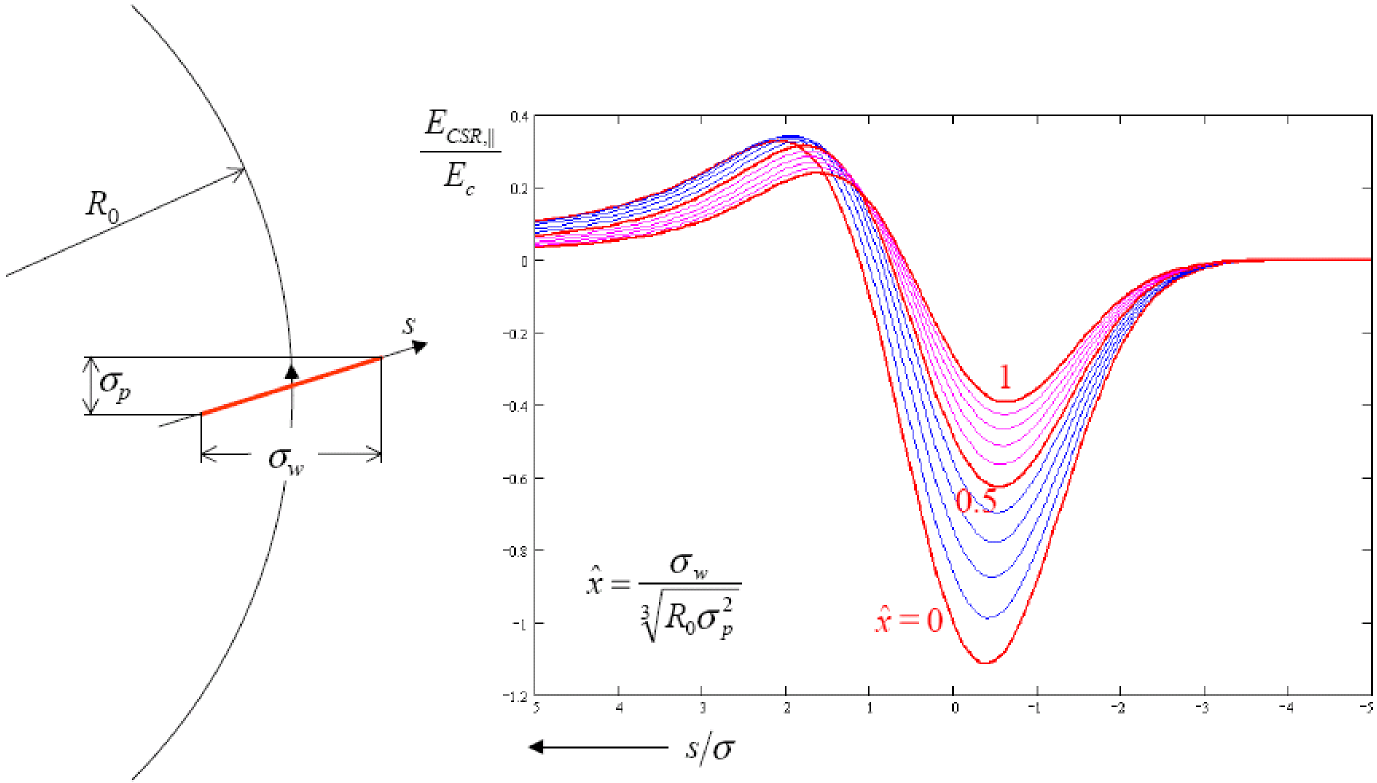
6. Summary of Key Results

- Delayed response of CSR force from bunch length variation
- Dependence of CSR force on particles’ transverse distribution around full compression
- The high frequency field signal generated around full compression and reaches the bunch shortly after
- Dohlus previous results can be explained as part of the delayed response

CSR Field of a Tilted Thin Beam



e.g. $R_0 = 10\,\mathrm{m}, \sigma_p = 100\,\mu\mathrm{m}, \sigma_w = 2\,\mathrm{mm} \rightarrow \hat{x} = 0.43$

u=-10.56/m (sc=1.2m), regular case

As in previous slide, for a given path length s, 2D CSR force on each particles in the bunch has different amplitude depending on the x and z coordinates of the particle. Here we plot the maximum and minimum CSR force on the bunch as a function of s. The average of CSR force on all the particles in the bunch indicates the energy loss by the bunch. The 2D results are in red and 1D results are in green, which is based on the rigid-bunch model with the same projected bunch length.